Section 2A

The Problem-Solving Power of Units

- The **units** of a quantity describe what is being measured or counted.

- We can add or subtract numbers only when they have the same units, but we can multiply and divide quantities with either the same or different units.

Reading Units

Operation	Key Word or Symbol	Example
Division	Per	miles ÷ hours is read as “miles per hour” or abbreviated mph
Raising to second power	Square	feet x feet or ft^2 read as “square feet” or “feet squared”
Raising to third power	Cube	feet x feet x feet or ft^3 read as “cubic feet” or “feet cubed”
Multiplication	Hyphen	kilowatts x hours written as “kilowatt-hours”

Identify the units of the following quantities. State the units mathematically and in words.

1. The unit price of oranges, found by dividing the price in dollars by the weight in pounds.

   - The unit price has units of dollars divided by pounds. This can be written mathematically as \$/lb and read as "dollars per pound."

2. The density of a rock, found by dividing its weight in grams by its volume in cubic centimeters.

   - The density has units of grams divided by cubic centimeters. This can be written mathematically as g/cm^3 and read as "grams per cubic centimeter" or "grams per centimeter cubed."

Working with Fractions

- We can express a fraction in three basic ways: common fraction form, decimal form, and percentage.
- Common fractions have the form a/b where a and b are real numbers and $b \neq 0$. The number on the top is called the **numerator** and the number on the bottom is called the **denominator**.
- A fraction is just another way of representing division
  - a/b means $a \div b$

Working with Fractions (addition/subtraction)

If two fractions have the same denominator, we can add (or subtract) the fractions by adding (or subtracting) the numerators and placing the result over the common denominator.

Ex: $$\frac{1}{8}+\frac{2}{8}=\frac{1+2}{8}=\frac{3}{8}$$

Ex: $$\frac{9}{11}-\frac{3}{11}=\frac{9-3}{11}=\frac{6}{11}$$

Working with Fractions (addition/subtraction)

- If the fractions do not have the same denominators, they must be rewritten as equivalent fractions with like denominators BEFORE they can be added or subtracted.

$$\frac{3}{4}+\frac{5}{6}$$

- Ex:

$$=\frac{9}{12}+\frac{10}{12}$$

$$=\frac{19}{12}$$

Working with Fractions (multiplication)

To multiply fractions, we multiply “straight across.”

This means that we multiply the numerators together and multiply the denominators together.

Ex: $\frac{2}{3} \times \frac{1}{4}$

$$= \frac{2 \times 1}{3 \times 4}$$

$$= \frac{1}{6}$$

Working with Fractions (division)

Two numbers are **reciprocals** if their product is 1.

Ex: $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals because $\frac{2}{3}\times\frac{3}{2}=1$

To divide fractions, we multiply by the reciprocal of the second fraction (or we "invert and multiply")

Ex: $\frac{5}{8}\div\frac{3}{4}=\frac{5}{8}\times\frac{4}{3}=\frac{5}{6}$

Conversion Factors

The statement 12 in. = 1 ft. is an example of a **conversion factor**.

This statement can be written in three equivalent ways:

12 in. = 1 ft.

$$\frac{12in}{1ft} = 1$$

$$\frac{1ft}{12in} = 1$$

Example: Find a conversion factor between square feet and square inches. Write it in three forms.

We know that 1 ft = 12 in.

$1\ ft^2 = 1ft \times 1ft$

$= 12in \times 12in$

$= 144\ in^2$

Thus, $1\,ft^2 = 144in^2$ and $\dfrac{1\,ft^2}{144in^2} = 1$ and $\dfrac{144in^2}{1\,ft^2} = 1$

Currency Conversions (Table 2.1 – page 87)

Currency	Dollars per Foreign	Foreign per Dollar
British pound	1.414	0.7072
Canadian dollar	0.7834	1.277
European euro	1.256	0.7965
Japanese yen	0.01007	99.34
Mexican peso	0.06584	15.19

Example: As you leave Paris, you convert 4500 euros to dollars. How many dollars do you receive?

From the table, we see that 1 euro = $1.256

$$4500 euro \times \frac{\$1.256}{1 euro} = \$5652$$

The price of 4500 euros is equivalent to $5652.

Example: There are approximately 3 million births in the United States each year. Find the birth rate in units of births per minute.

We are asked to find the birth rate per minute. Recall that "per" means division.

We are given that there are 3 million births per year, so

$$\frac{3{,}000{,}000 \text{ births}}{1 \text{ year}}$$

We need to convert years to minutes as asked for in the problem.

$$\frac{3{,}000{,}000 \text{ births}}{1 \text{ year}} \times \frac{1 \text{ year}}{365 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ min}}$$

$$= \frac{5.707762557 \text{ births}}{\text{min}}$$

There are 6 births per minute.