Section 3A

Uses and Abuses of Percentages

Three Ways of Using Percentages

1. Using Percentages as Fractions

   Percent is just another way of saying "divided by 100," so P% means P/100.

2. Using Percentages to Describe Change

   The **absolute change** describes the actual increase or decrease from a reference value to a new value.

   absolute change = new value – reference value

   The **relative change** is a fraction that describes the size of the absolute change in comparison to the reference value.

$$\text{relative change} = \frac{\text{absolute change}}{\text{reference value}} = \frac{\text{new value} - \text{reference value}}{\text{reference value}}$$

Three Ways of Using Percentages

3. Using Percentages for Comparisons

   The **absolute difference** is the actual difference between the compared value and the reference value.

   absolute difference = compared value – reference value

   The **relative difference** describes the size of the absolute difference as a fraction of the reference value.

$$\text{relative difference} = \frac{\text{absolute difference}}{\text{reference value}}$$

Example: Express the first number as a percentage of the second number: 28 pounds of recyclable trash in a barrel of 52 pounds of trash.

$$\frac{28lb}{52lb} = .538 = 53.8\%$$

The population of the United States increased from 249 million in 1990 to 308 million in 2010. Find the absolute change and the percentage change.

Absolute change = new value – reference value

= 308 million – 249 million

= 59 million

Percentage change = $\frac{\text{absolute change}}{\text{reference value}}$

$$= \frac{59 \text{ million}}{249 \text{ million}}$$

$$= .237$$

$$= 23.7\%$$

Example: The gestation period of humans (266 days) is _______ percent longer than the gestation period of grizzly bears (220 days).

Relative difference = $\frac{\text{absolute difference}}{\text{reference value}}$

$$= \frac{266 - 220}{220}$$

$$= \frac{46}{220}$$

$$= .209$$

$$= 20.9\%$$

OF Versus MORE THAN (or LESS THAN)

If the compared value is P% **more than** the reference value, it is (100 + P)% of the reference value.

If the compared value is P% **less than** the reference value, it is (100 – P)% of the reference value.

Example: Will is 22% taller than Wanda, so Will's height is ____% of Wanda's height.

(100 + 22)% = 122 %

Percentages of Percentages

- When you see a change or difference in **percentage points**, you can assume it is an **absolute** change or difference.

- A change with the % sign or the word **percent** is a **relative** change or difference.

Example: The annual interest rate for Jack's savings account increased from 2.3% to 2.8%. Describe as an absolute change in terms of percentage points and as a relative change in terms of a percentage.

Absolute change = new value – reference value

=2.8% - 2.3%

=0.5%

Relative change = $\frac{\text{absolute change}}{\text{reference value}}$

$$= \frac{0.5\%}{2.3\%}$$

$$= 0.217$$

$$= 21.7\%$$

Abuses of Percentages

1. Beware of Shifting Reference Values

   Ex: Decide if the following is true or false and explain: You receive a pay raise of 5%, then receive a pay cut of 5%. After the two changes, your salary is unchanged.

   Let's assume a starting salary of $1000. After pay raise, your salary is $1000 + 0.05*1000 = $1050.

   Then your 5% pay cut gives you $1050 – 0.05*$1050 = $997.50.

   False. You end up with less money than you started with.

Abuses of Percentages

2. Less Than Nothing

   Ex: Decide whether the claim could be true: By turning off her lights and closing her windows at night, Maria saved 120% on her monthly energy bill.

   If she saved 100%, her bill would be $0. If she saved 120%, her bill would be less than $0, they would owe her money. This claim could not be true.

Abuses of Percentages

3. Don't Average Percentages

   Ex: A player has a batting average over many games of 0.400. In his next game, he goes 2 for 4, which is a batting average of 0.500 for the game. Does it follow that his new batting average is (0.400 + 0.500)/2 = 0.450? Explain.

   No. Suppose the initial batting average of 0.400 is from 500 at-bats. This means that he got hits 40% of his 500 at-bats or 0.40*500 = 200 hits. When he goes 2 for 4 in the next game, his batting average become (200 + 2) / (500 + 4) = 202/504 = 0.401.