Section 8A

Growth: Linear versus Exponential

- **<u>Linear growth</u>** occurs when a quantity grows by some fixed *absolute* amount in each unit of time.

- **<u>Exponential growth</u>** occurs when a quantity grows by the same fixed *relative* amount – that is, by the same percentage – in each unit of time.

Example 1: State whether the growth (or decay) is linear or exponential. Then answer the associated question: The population of Winesburg is increasing at a rate of 4% per year. If the population is 75,000 today, what will it be in 3 years?

- Since the population is growing by a percentage (*relative*) amount each year, this is **exponential growth**.
- Year 1: The population grows by 4% of 75,000. The new population is 75,000 + 0.04*75,000 = 78,000.
- Year 2: The population grows by 4% of 78,000. The new population is 78,000 + 0.04*78,000 = 81,120.
- Year 3: The population grows by 4% of 81,120. The new population is 81,120 + 0.04*81,120 = 84,365.

- Thus, the population of Winesburg after 3 years is **84,365**.

Example 2: State whether the growth (or decay) is linear or exponential. Then answer the associated question: The price of gasoline is increasing by 3 cents per week. If the price of gasoline is $3.10 per gallon today, what will it be in ten weeks?

- The price of gasoline increases by an *absolute* amount each week, so this is **linear growth.**

- Since the price of gasoline increases by 3 cents **each** week, in ten weeks it increases by $0.03 * 10 = $0.30

- The price of gasoline in ten weeks will be $3.10 + 0.30 = **$3.40**

Hero to Headless

- A king was so enchanted when the game of chess was invented that he told the inventor to "name his reward."

- The inventor told the king to put one grain of wheat on the first square, two grains on the second square, four grains on the third square, etc. and the king gladly agreed.

- Let's see how many grains of wheat there will be on the whole chessboard.

Hero to Headless

Square	Grains on This Square	Total Grains So Far	Formula for Total Grains
1	1	1	
2	2	1+2=3	
3	4	3+4=7	
4	8	7+8=15	
5	16	15+16=31	
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The Magic Penny

- A leprechaun promises you great wealth and gives you a penny. He tells you to put it under your pillow.

- The next morning, you find two pennies.

- The following morning, you find four pennies.

- How much money will you have after 30 days?

The Magic Penny

Day	Amount Under Pillow	Formula
0	$0.01	
1	$0.02	
2	$0.04	
3	$0.08	
4	$0.16	
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Key Facts about Exponential Growth

- **Exponential growth** leads to repeated doublings. With each doubling, the amount of increase is approximately equal to the sum of all preceding doublings.

- **Exponential growth** cannot continue indefinitely. After only a relatively small number of doublings, exponentially growing quantities reach impossible proportions.