Section 4B

The Power of Compounding

- The **principal** is the balance upon which interest is paid.

- **Simple interest** is interest paid *only* on the original principal, and not on any interest added at later dates.

Simple Interest Formulas

- $I = P \cdot APR \cdot Y$ is used to calculate the amount of interest earned.

- $A = P(1+APR \cdot Y)$ is used to calculate the account balance after a specified period of time.

- In each of these formulas,
- I = interest earned
- P = principal
- APR = annual percentage rate (as a decimal)
- Y = number of years

Example 1: You deposit $1200 in an account that earns 2% simple interest. How much money will you have after five years? How much interest did the account earn?

- Since this is a simple interest account and we want to find the account balance, we will use the formula: A = P(1+APR·Y).
- We know: P = $1200; APR = 0.02; Y = 5
- A = $1200(1+0.02*5)
- A = **$1320**

- We can find the interest earned by using the formula I = P·APR·Y.
- So I = $1200*0.02*5 = $120
- OR we can find the interest by subtracting the principal from the account balance: I = $1320 – 1200 = $120

- **Compound interest** is interest paid on *both* the original principal and on all interest that has been added to the original principal.

- It is paid on the current account balance.

Compound Interest Formula

$$A = P\left(1 + \frac{APR}{n}\right)^{(nY)}$$

- A = accumulated balance
- APR = annual percentage rate (as a decimal)
- n = number of compoundings per year
- Y = number of years
- P = original principal

Example 2: Compute the balance when $15,000 is invested at an annual rate of 3.25%, compounded quarterly, for 8 years. How much interest is earned?

- We know: P = $15,000; APR = 0.0325; Y = 8; n = 4
- We use the compound interest formula to find the account balance:

$$A = \$15000\left(1+\frac{0.0325}{4}\right)^{(4*8)}$$

- After 8 years, there will be **$19,433.52** in the account
- We can find the amount of interest earned by subtracting the principal from the account balance.
- I = $19,433.52 – 15,000 = **$4,433.52**

- The **annual percentage yield (APY)** is the actual percentage by which a balance increases in *one* year.

- It is sometimes referred to as the **effective yield** or simply the **yield**.

- APY = relative increase = $\dfrac{\text{absolute increase}}{\text{starting principal}} = \dfrac{A - P}{P}$

Example 3: Calculate the APY for an account with an APR of 4.2%, compounded daily.

- First, we need to calculate an amount of increase. Choose a value for P, we'll use P = $1000.
- We know: APR = 0.042, Y = 1, and n = 365

$$A = \$1000\left(1+\frac{0.042}{365}\right)^{(365*1)}$$

- So, A = $1042.89
- Thus,

$$APY = \frac{\$1042.89-\$1000}{\$1000} = \frac{\$42.89}{\$1000} = 0.04289 = 4.289\%$$

APR vs. APY

- APR = annual percentage rate
- APY = annual percentage yield

- APY = APR if interest is compounded annually.

- APY>APR if interest is compounded more than once a year.

Continuous Compounding

- Compounding infinitely many times per year is called **continuous compounding**.
- Since we can't count number of compoundings per year, we need a special formula.

$$A = P * e^{(APR*Y)}$$

- A = accumulated balance
- P = principal
- APR = annual percentage rate (as a decimal)
- Y = number of years
- e = a special irrational number with a value of e≈2.71828 (built into your calculator)

Example 4: Calculate the account balance if you deposit $7,000 in an account with an APR of 6.7%, compounded continuously, for 20 years.

- We know: P = $7000; APR = 0.067; Y = 20

$$A = \$7000 * e^{(0.067*20)}$$

- The account balance is **$26,733.30**