

Section 4C

Savings Plans and Investments

- A **savings plan** is a long-term account where the investor deposits smaller amounts on a regular basis.
- These are also called **annuities** because the investor is making a series of equal, regular payments into an account.
- Each payment receives compound interest.
- Some examples of savings plans include: Individual Retirement Accounts (IRAs), 401(k) plans, Keogh plans, and employee pension plans
- The accumulated balance (A) is often called the *future value* (FV)

Savings Plan Formula

$$A = PMT * \frac{\left(\left(1 + \frac{APR}{n} \right)^{(nY)} - 1 \right)}{\left(\frac{APR}{n} \right)}$$

where

- A = accumulated balance
- PMT = regular payment (deposit) amount
- APR = annual percentage rate (as a decimal)
- n = number of payment periods per year
- Y = number of years

Example 1: Find the savings plan balance after 2 years with an APR of 5% and monthly payments of \$250.

- We know: $PMT = \$250$; $APR = 0.05$; $Y = 2$; $n = 12$

$$A = \$250 * \frac{\left(\left(1 + \frac{0.05}{12} \right)^{(12*2)} - 1 \right)}{\left(\frac{0.05}{12} \right)}$$

- The savings plan balance after 2 years is **\$6296.48**

Example 2: A friend creates an IRA with an APR of 6.25%. She starts the IRA at age 25 and deposits \$50 per month. How much will her IRA contain when she retires at age 65? How much interest did her account earn?

- We know: APR = 0.0625; PMT = \$50; n = 12; Y = 40

$$A = \$50 \frac{\left(\left(1 + \frac{0.0625}{12} \right)^{(12*40)} - 1 \right)}{\left(\frac{0.0625}{12} \right)}$$

- She will have **\$106,595.63** when she retires.
- To find the amount of interest, we must first find how much money she contributed to the IRA.
- She contributed: $\$50 * 12 * 40 = \$24,000$
- The rest of the account balance is interest, so her IRA earned $\$106,595.63 - 24,000 = \mathbf{\$82,595.63}$ in interest.

Planning Ahead

- We can use our savings plan formula to plan ahead. If we know how much we would like to have in the long-run, we can find out how much our periodic deposits would need to be.
- We will use the savings plan formula solved for PMT

$$PMT = \frac{A * \left(\frac{APR}{n} \right)}{\left(\left(1 + \frac{APR}{n} \right)^{(n*Y)} - 1 \right)}$$

Example 3: At age 20 when you graduate, you start saving for retirement. If your investment plan pays an APR of 4.5% and you want to have \$5 million when you retire in 45 years, how much should you deposit monthly?

- We know: $APR = 0.045$; $A = \$5,000,000$; $n = 12$; $Y = 45$

$$PMT = \frac{\$5,000,000 * \left(\frac{0.045}{12} \right)}{\left(\left(1 + \frac{0.045}{12} \right)^{(12*45)} - 1 \right)}$$

- We would need to deposit **\$2,863.70** per month.

Example 4: Suppose you are 25 years old and would like to retire at age 65. Furthermore, you would like to have a retirement fund from which you can draw an income of \$200,000 per year – forever! How can you do it? Assume a constant APR of 6%.

- We want our retirement fund to be large enough so that we are earning \$200,000 per year in *interest*.
- We need to find out what balance we would need to earn \$200,000 annually from interest.
- Since we have a constant APR of 6%, the \$200,000 must be 6% of the total balance.
- That is, $\$200,000 = 0.06 * (\text{total balance})$
- Total balance = $\$200,000/0.06 \approx \$3,333,334$
- Now we can calculate the required payment to end with a balance of \$3,333,334

$$PMT = \frac{\$3,333,334 * \left(\frac{0.06}{12}\right)}{\left(\left(1 + \frac{0.06}{12}\right)^{(12*40)} - 1\right)} = \$1673.79$$

- In the previous examples, we've assumed that you receive a constant interest rate for a long period of time. In reality, interest rates usually vary over time.
- Consider an investment that grows from an original principal P to a later accumulated balance A .
- The **total return** is the relative change in the investment value.
- Total return = $\frac{(A - P)}{P}$
- The **annual return** is the annual percentage yield (APY) that would give the same overall growth.
- Annual return = $\left(\frac{A}{P}\right)^{(1/Y)} - 1$
- where Y is the investment period in years

Example 5: Twenty years after purchasing shares in a mutual fund for \$6500, you sell them for \$11,300. Find the total and annual return.

- We know: $A = \$11,300$; $P = \$6500$; $Y = 20$

$$\frac{(A - P)}{P} = \frac{(\$11300 - \$6500)}{\$6500}$$

- The total return is **$0.7384615385 = 73.84615385\%$**

$$\left(\frac{A}{P}\right)^{(1/Y)} - 1 = \left(\frac{\$11300}{\$6500}\right)^{(1/20)} - 1$$

- The annual return is **$0.0280358371 = 2.80358371\%$**

Example 6: Five years after paying \$5000 for shares in a new company, you sell the shares for \$3000 (at a loss). Find the total and annual return.

- We know $A = \$3000$; $P = \$5000$; $Y = 5$

$$\frac{(A - P)}{P} = \frac{(\$3000 - \$5000)}{\$5000}$$

- The total return is **$-.4 = -40\%$**

$$\left(\frac{A}{P}\right)^{(1/Y)} - 1 = \left(\frac{\$3000}{\$5000}\right)^{(1/5)} - 1$$

- The annual return is **$-0.0971195486 = -9.71195486\%$**

Types of Investments

- **Stock** (or *equity*) gives you a share of ownership in a company. You invest some principal amount to purchase the stock, and the only way to get your money out is to sell the stock. Because stock prices change with time, the sale may give you either a gain or a loss on your investment.
- A **bond** (or *debt*) represents a promise of future cash. You buy a bond by paying some principal amount to the issuing government or corporation. The issuer pays you simple interest (as opposed to compound interest) and promises to pay back your principal at some later date.

Types of Investments

- **Cash** investments include money you deposit into bank accounts, certificates of deposit (CDs), and U.S. Treasury bills (T-bills). Cash investments generally earn interest.
- You can invest in either of these either *directly* or *indirectly*.
- To invest *directly*, you would buy the individual investments yourself.
- To invest *indirectly*, you can purchase shares in a **mutual fund**, where a professional fund manager invests your money (and the money of others participating in the fund).

Investment Considerations

- **Liquidity** – How difficult is it to take out your money?
- **Risk** – Is your investment principal at risk? Is there a possibility that your investment can drop in value?
- **Return** – How much return (total or annual) can you expect on your investment? A higher return means you earn more money. Generally, low-risk investments offer relatively low returns, while high-risk investments offer the prospects of higher returns – along with the possibility of losing your principal.

Stocks

- Two ways to make money on stocks:
- 1. You can make money if you sell a stock for more than you paid for it. This indicates a **capital gain** on the sale of the stock.
You can also lose money if you sell a stock for less than you paid for it. This indicates a **capital loss** on the sale of the stock.
- 2. You can make money while you own the stock if the corporation distributes part or all of its profits to stockholders as **dividends**. Each share of stock is paid the same dividend, so the amount of money you receive depends on the number of shares you own.
Not all companies distribute profits as dividends, some reinvest all profits within the corporation.

Stocks (see page 239)

- **Stock/symbol** – the company name and the “ticker symbol” used to identify it
- **Last** – price per share, in dollars, at time shown
- **Change** – change in share price, in dollars, since prior day’s close
- **% Change** – percentage change in share price since prior day’s close
- **Open/High/Low** – opening, highest, and lowest share prices so far today
- **52-Week High/Low** – highest and lowest share prices during the past 52 weeks
- **Volume** – the number of shares that have been traded today
- **Market Cap** – total stock value of the company; equal to the total number of outstanding shares * the share price
- **Price-to-Earnings Ratio (P/E)** – the share price divided by the earnings (profit) per share over the past year
- **Dividend(latest quarter)** – the dividend paid during the last quarter, per share
- **Shares Outstanding** – the total number of shares that exist for the company
- **Dividend Yield** – the annual yield of the dividend based on current share price and dividend per share

Bonds

- The **face value** (or *par value*) of the bond is the price you must pay the issuer to buy it at the time it is issued.
- The **coupon rate** of the bond is the *simple interest rate* that the issuer promises to pay.
- The **maturity date** of the bond is the date on which the issuer promises to repay the face value of the bond.
- Bonds can also be bought and sold after they are issued, in the *secondary bond market*.
- **Current yield** of a bond is the amount of interest it pays each year divided by the bond's current price (*not its face value*).