Section 6D

Statistical Inference

Ex: You flip a coin 3000 times and get 2999 heads. Do you think the coin is a fair coin?

- Probably not. We would expect roughly ½ heads and ½ tails and our results are not even close.

Ex: You flip a coin 3000 times and get 1550 heads. Do you think the coin is a fair coin?

- Reasonable since we expect ½ heads and ½ tails.

A set of measurements or observations in a statistical study is said to be **statistically significant** if it is unlikely to have occurred by chance.

Idea of statistical significance can be applied to making an inference from a sample to a population.

We need a quantitative definition for statistical significance when the findings are less obvious.

- If the probability of an observed difference occurring by chance is 1 in 20 (0.05) or less, the difference is significant **at the 0.05 level**.

- If the probability of an observed difference occurring by chance is 1 in 100 (0.01) or less, the difference is significant **at the 0.01 level**.

A study that measured the body temperatures of 106 individuals found a mean of 98.20˚F. The accepted value for human body temperature is 98.60˚F. If we assume that the mean body temperature is actually 98.60˚F, the probability of getting a sample with a mean of 98.20˚F or less turns out to be less than 1 in 1 million. Is this result significant at the 0.05 level? At the 0.01 level?

1 in 1 million is less than both 1 in 20 and 1 in 100, so it would be significant **at both the 0.05 and 0.01 levels**.

Central Limit Theorem: The distribution of proportions from many samples of the same size is approximately a normal distribution. The mean of this distribution is the true proportion for the entire population.

Sampling distribution – consists of proportions from many individual samples

Suppose you draw a single sample of size n from a large population and measure its sample proportion. The **margin of error** for 95% confidence is

$$\text{margin of error} \approx \frac{1}{\sqrt{n}}$$

The 95% **confidence interval** is found by subtracting and adding the margin of error from the sample proportion.

You can be 95% confident that the true population proportion lies within this interval.

The margin of error decreases as the sample size increases.

A CBS news poll of 1003 American adults concluded that 48% of Americans feel that people with strong religious convictions face discrimination in this country. Find the margin of error and the 95% confidence interval.

Note that our sample size is n = 1003.

$$\text{Then, the margin of error} = \frac{1}{\sqrt{1003}}$$

$$= 0.0315754489$$

$$= 3\%$$

The 95% confidence interval is from (48 – 3)% to (48 + 3)% which is **45% to 51%**.

Hypothesis testing – the techniques for testing the validity of claims

Null hypothesis – claims a specific value for a population parameter.

- It is often the value expected in the case of no special effect.

- Takes the form

null hypothesis: Population parameter = claimed value

Alternative hypothesis – the claim that is accepted if the null hypothesis is rejected

Outcomes of a Hypothesis Test

We assume the null hypothesis is true until proven otherwise.

There are two possible outcomes to any hypothesis test.

1. **Reject the null hypothesis** – have evidence that supports the alternative hypothesis

2. **Not reject the null hypothesis** – lack sufficient evidence to support the alternative hypothesis

We cannot prove that a null hypothesis is *true*, which is why "accepting the null hypothesis" is not a possible outcome.

We can find evidence that the null hypothesis is *false*, which would cause us to reject it.

The governor claims that the percentage of adults over 25 who have graduated from high school is greater than 85%, the national average. State the null and alternative hypotheses and describe the two possible outcomes in the context of the situation.

Null hypothesis: percentage of high school graduates = 85%

Alternative hypothesis: percentage of high school graduates > 85%

Rejecting the null hypothesis means that there is evidence that the percentage of high school graduates **is** greater than 85%

Failing to reject the null hypothesis means that there is insufficient evidence to conclude that the percentage of high school graduates is greater than 85%

Hypothesis Test Decisions

We decide the outcome of a hypothesis test by comparing the actual sample result to the result expected if the null hypothesis is true.

- If the chance of a sample result at least as extreme as the observed result is less than 1 in 100 (0.01), the test is significant at the 0.01 level. The test offers *strong* evidence for **rejecting the null hypothesis**.

- If the chance of a sample result at least as extreme as the observed result is less than 1 in 20 (0.05), the test is significant at the 0.05 level. The test offers *moderate* evidence for **rejecting the null hypothesis**.

- If the chance of a sample result at least as extreme as the observed result is greater than 1 in 20, the test is not significant. The test does **not provide sufficient grounds for rejecting the null hypothesis**.

A Senate candidate claims that a majority of voters support her. A poll of 400 voters finds that the proportion of voters who support the candidate is 0.51 (51%). Assuming that the proportion of people in the population who support her is p = 0.5, the probability of selecting a sample in which the proportion is 0.51 or more is 0.345. Formulate the null and alternative hypotheses and discuss whether the sample provides evidence for rejecting or not rejecting the null hypothesis.

Null hypothesis: proportion of supporting voters = 0.5

Alternative hypothesis: proportion of supporting voters > 0.5

The problem says the probability of a sample where the proportion is > 0.5 is 0.345. This is greater than 0.05, so the result is not significant at the 0.05 level.

There is not sufficient evidence for rejecting the null hypothesis (fail to reject).