Sections 7A & 7B

Fundamentals of Probability

&

Combining Probabilities

Outcomes are the most basic possible results of observations or experiments

An **event** consists of one or more outcomes that share a property of interest.

You flip two fair coins. List the outcomes. List the events in terms of "number of heads."

Outcomes = {HH, HT, TH, TT}

Events = 0H, 1H, 2H

List the outcomes for a family choosing to have 3 children.

They could have all girls – GGG

They could have all boys – BBB

They could have 1 girl and 2 boys – GBB, BGB, or BBG

They could have 2 girls and 1 boy – GGB, GBG, or BGG.

Outcomes = {GGG, BBB, GBB, BGB, BBG, GGB, GBG, BGG}

The **probability of an event**, P(event), is always a number between 0 and 1.

P(event) = 0 means that the event is **impossible**.

P(event) = 1 means that the event is **certain**.

A **theoretical probability** is based on a model in which all outcomes are equally likely.

- Based on what we expect to happen (in theory)

An **empirical probability** is based on actual observations or experiments.

A **subjective probability** is an estimate based on experience or intuition.

Probability

$$P(A) = \frac{\text{\# of ways A can occur}}{\text{total \# of outcomes}}$$

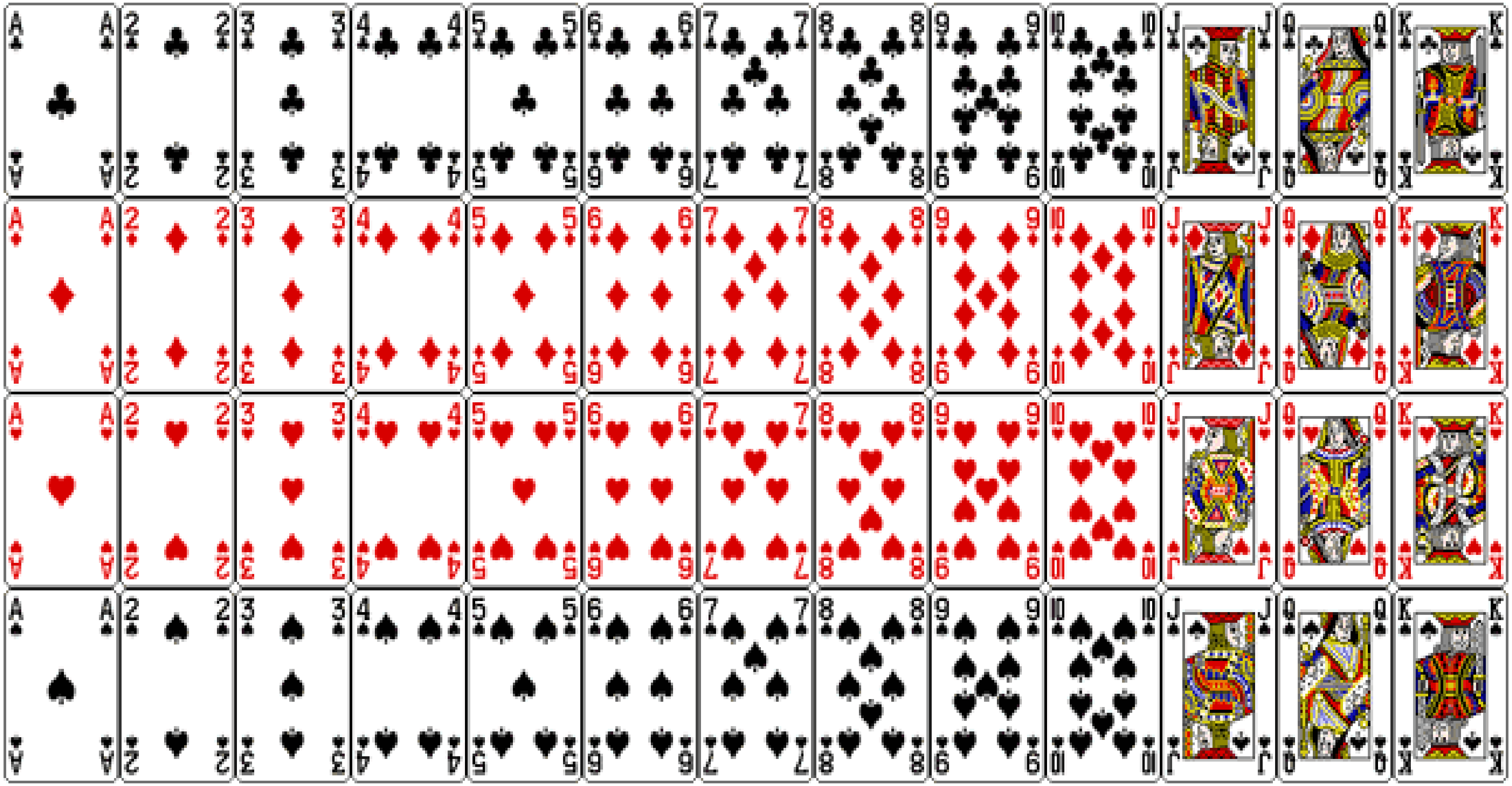
Calculate the probability of having at least 2 boys if a couple has 3 children.

Recall from earlier,

outcomes = {GGG, BBB, GBB, BGB, BBG, GGB, GBG, BGG}

There are 4 outcomes that have at least two boys out of the 8 possible outcomes.

Thus, P(at least 2 boys) = 4/8 = ½ = 0.5



Find the probability of drawing a queen from a standard deck of cards.

There are 52 cards in a standard deck.

Of the 52 cards, 4 of them are queens.

P(Queen) = 4/52 or 1/13

= 0.0769230769

You count 42 heads when you toss a coin 100 times. If you don't know whether the coin is fair, what is the probability that the next toss will be a tail?

If you have 42 heads, that means that you had 100 – 42 = 58 tails.

P (tail) = 58/100 or 29/50

= 0.58

Total # of outcomes

Sometimes we have a hard time counting the total # of outcomes, especially when it is large.

If we just need to know the total # of outcomes we can use other methods that don't involve listing all the outcomes.

Multiplication Principle

Suppose there are M possible outcomes for one process and N possible outcomes for a second process.

The total # of outcomes for the two processes combined is M x N.

This extends to any number of processes.

Count the number of possible outcomes for a couple who has three children.

$$2 \times 2 \times 2 = 8$$

Count the number of possible outcomes if you flip two fair coins.

$$2 \times 2 = 4$$

A restaurant has a special menu that features three choices of soup, six choices of entrée, and five choices of dessert. How many different three-course meals could you order?

3 soups x 6 entrees x 5 desserts

= 90 different three-course meals

The probability that event A does **not** occur is given by 1 – P(A).

We saw earlier that P(at least 2 boys) = ½.

So, the P(less than 2 boys) = 1 – ½ = ½

A **probability distribution** represents the probabilities of all possible events (usually in tabular form).

Make a probability distribution for the number of heads that occur when two coins are flipped.

Recall, outcomes = {HH, HT, TH, TT}

Event (# of heads)	Probability
0	¼
1	2/4 = 1/2
2	1/4

Consider rolling 2 dice and looking at the sum. The table below gives the possible outcomes. (sums are listed in black)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Odds

The **odds for** an event A are

$$\text{odds for event A} = \frac{P(A)}{P(\text{not } A)}$$

The **odds against** an event A are

$$\text{odds against event A} = \frac{P(\text{not } A)}{P(A)}$$

Two events are **<u>independent</u>** if the outcome of one event **does not** affect the probability of the other event.

For independent events A & B,

P(A and B) = P(A) x P(B).

Two events are **<u>dependent</u>** if the outcome of one event **does** affect the probability of the other event.

For dependent events A & B,

P(A and B) = P(A) x P(B given A)

where P(B given A) means the probability of event B given the occurrence of event A.

Find the probability of drawing 3 queens from a standard deck if the card is returned to the deck each time.

These 3 events will be independent. Since we are returning the card to the deck after each draw, the outcome of one draw does not affect the probability for the subsequent draws.

P(3 queens) = P(queen & queen & queen)

= P(queen) x P(queen) x P(queen)

= 4/52 x 4/52 x 4/52

= 64/140608

= 0.0004551661356

Find the probability of drawing 3 queens from a standard deck if the card is not returned to the deck each time.

Since the card is NOT returned each time, the outcome of one draw does affect the probability of the next draw. Thus these events are dependent.

P(3 queens) = P(queen & queen & queen)

= 4/52 x 3/51 x 2/50

= 24/132600

= 0.0001809954751

Two events are **non-overlapping** if they **cannot** occur together.

If A & B are non-overlapping,

$$P(A \text{ or } B) = P(A) + P(B)$$

Two events are **overlapping** if they **can** occur together.

If A & B are overlapping,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Find the probability of drawing either a jack or a club from a standard deck of cards.

The events “jack” and “club” are overlapping events since you can have a card that is both a jack and a club.

$$
\begin{aligned}
P(\text{jack or club}) &= P(\text{jack}) + P(\text{club}) - P(\text{jack \& club}) \\
&= 4/52 + 13/52 - 1/52 \\
&= 16/52 \ (\text{or } 4/13) \\
&= 0.3076923077
\end{aligned}
$$

Find the probability of drawing either a jack or a queen from a standard deck of cards.

The events "jack" and "queen" are non-overlapping events since you cannot have a card that is both a jack and a queen.

P(jack or queen) = P(jack) + P(queen)

= 4/52 + 4/52

= 8/52 (or 2/13)

= 0.1538461538

At Least Once Rule (for independent events)

Suppose the probability of an event A occurring in one trial is P(A).

If all trials are independent, the probability that event A occurs **at least once** in n trials is given by:

$$P(\text{at least one event A in n trials}) = 1 - P(\text{no events A in n trials})$$
$$= 1 - [P(\text{not A in one trial})]^n$$

Find the probability of getting at least one 5 when rolling three fair dice.

P(rolling a 5) = 1/6, so P(not rolling a 5) = 5/6

P(at least one 5 in 3 trials)

$= 1 – [P(\text{no 5 in one trial})]^3$

$= 1 – (5/6)^3$

= 0.4212962963