Section 7C

The Law of Large Numbers

The Law of Large Numbers

If a process is repeated over many trials, the proportion of the trials in which event A occurs will be close to the probability P(A). The larger the number of trials, the closer the proportion should be to P(A).

Says that if the trial is conducted many times, the empirical probability will get closer and closer to the theoretical probability.

Expected Value

Expected value is a long-term average.

Expected value =
(value of event A) x P(A) + (value of event B) x P(B)

This formula can be extended for more events.

You are given 9 to 1 odds against tossing three heads with three coins, meaning you win $9 if you succeed and you lose $1 if you fail. Find the expected value (to you). Would you expect to win or lose money in 1 game? In 100 games?

When you toss three coins, there are 8 possible outcomes.

P(3 H) = 1/8 and P(not 3 H) = 7/8

Expected value = (value of win) x P(win) + (value of loss) x P(loss)

= $9 x 1/8 + (- $1) x 7/8

= $0.25

The outcome of 1 game cannot be predicted, but we would expect to win money over 100 games.

An insurance policy sells for $900. Based on past data, an average of 1 in 50 policyholders will file a $10,000 claim, an average of 1 in 100 policyholders will file a $25,000 claim, and an average of 1 in 250 policyholders will file a $60,000 claim. If the company sells 10,000 policies, what is the expected profit or loss?

Expected value = $900 x 1 + (- $10,000) x 1/50 +

(- $25,000) x 1/100 + (- $60,000) x 1/250

= $210

This means that on average the company expects a $210 profit per policy.

Since they sold 10,000 policies, that means they have a profit of $210 x 10,000 = $2,100,000.

The Gambler's Fallacy

The **gambler's fallacy** is the mistaken belief that a streak of bad luck makes a person "due" for a streak of good luck.

Even though if you play "long enough" you will eventually win a game, you have to consider the continued losses that had been occurring.

If you spend more money than what you won, you did not win at all.

The **house edge** is the expected value *to the casino* of a particular bet.

A casino makes money because games are set up so that the expected earnings of players are negative, which means the casino's earnings are positive.