

Section 11C

Proportion and the Golden Ratio

How can a line segment be divided into two pieces that have the most appeal and beauty?

The Greeks claimed that the most visually pleasing division of the line had the property that the ratio of the length of the long piece to the length of the short piece is the same as the ratio of the length of the entire line segment to the length of the long piece.

That is, $\frac{L}{1} = \frac{L+1}{L}$ where L is the length of the long piece and 1 is the length of the short piece.

The previous equation can be solved to find that L has a special value, denoted by the Greek letter Φ , which is

$$\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61803 \approx \frac{8}{5}$$

The number Φ is commonly called the **golden ratio** or the **divine proportion**.

A line is subdivided according to the golden ratio, with the smaller piece having a length of 5 meters. What is the length of the entire line?

Since the length of the smaller piece is 5 meters, the longer piece is

$$= 5 \times \Phi$$

$$= 5 \times 1.61803$$

$$= 8.09015 \text{ meters}$$

The entire line would have a length of

$$8.09015 \text{ m} + 5 \text{ m}$$

$$= 13.09015 \text{ meters}$$

A **golden rectangle** is any rectangle whose long side is Φ times as long as its short side.

- length and width have a ratio of Φ

The golden rectangle (or a rectangle close to a golden rectangle) appears in many places – architecture, artwork, photographs, note cards, cereal boxes, windows, nature, etc.

A golden rectangle has one side with a length of 12.6 feet. Find the length of the other side. Use $\Phi = 1.62$ for your work.

The problem does not tell us whether the 12.6 feet indicates the length of the long or short side, so we will have to find the other side in both cases.

If 12.6 feet is the shorter side, the longer side is

$$\begin{aligned} & \Phi \times 12.6 \text{ feet} \\ &= 1.62 \times 12.6 \text{ feet} \\ &= 20.412 \text{ feet} \end{aligned}$$

If 12.6 feet is the longer side, the shorter side is

$$\begin{aligned} & 12.6 \text{ feet} \div \Phi \\ &= 12.6 \text{ feet} \div 1.62 \\ &= 7.778 \text{ feet} \end{aligned}$$

The other side could be either 20.412 feet or 7.778 feet to still have a golden rectangle.

Fibonacci Sequence

The sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, ...
is called the **Fibonacci sequence**.

Notice that each term is found by adding the two
terms before it.